Tutorial 7. Math 2010 B

Outline

- Revision (Week 1-6)

Tutors (HWS)

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One remark:

HW 2. Find all solutions of the one-dimensional heat equation of the form ....

In general, Given an one dimensional heat equation.

the solution will be not unique except giving initial condition

& boundary condition.

Q: Let L be the x-axis in $\mathbb{R}^3$

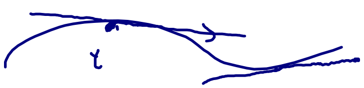
Define $\vec{r}: \mathbb{R} \longrightarrow \mathbb{R}^3$ by

$$\vec{r}(t) = (0, t, e^t), \quad t \in \mathbb{R}$$

1) Find the tangent line of $\vec{r}$ at $\tau \in \mathbb{R}$.

2) Find the distance between L and L_0

Sol 1): line is determined by



2 points or one point + direct vector.

$$r'(t) = (0, 1, e^t) \qquad r(t) = (0, t, e^t)$$

↑ direction

← one point.

$$\Rightarrow \quad L_t = \{ (0, t+s, (1+s)e^t) \in \mathbb{R}^3, \ s \in \mathbb{R} \}$$

In particular: $L_0 = \{ (0, s, 1+s) \in \mathbb{R}^3, \ s \in \mathbb{R} \}$

2) distance L an L_0

Let A be a point on L_0, and B be a point on L.

s.t $\overrightarrow{AB} \perp L$ and $\overrightarrow{AB} \perp L_0$

step ① $\begin{cases} A = (0,\ a,\ 1+a) & \text{for some } a \in \mathbb{R} \\ B = (b,\ 0,\ 0) & \text{for some } b \in \mathbb{R} \end{cases}$

Step 1 1. A. B

Step 2 $\overrightarrow{AB} \perp L$, and $\overrightarrow{AB} \perp L_0$

$\Downarrow$

A. B.

Step 3: $|AB|$

$\overrightarrow{AB} = (-b,\ a,\ 1+a)$

$\overrightarrow{AB} \perp L \iff \overrightarrow{AB} \cdot \vec{L} = 0$

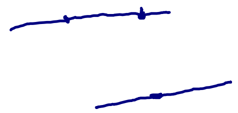
$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -b \\ a \\ 1+a \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

On solving $a = -\frac{1}{2}$ and $b = 0$.

The distance betwe L and L_0 $= \sqrt{0^2 + (-\frac{1}{2})^2 + (1-\frac{1}{2})^2} = \frac{1}{\sqrt{2}}$

on line / on line

Line : two points. or one point + one direction



plane : two method.

① Equation. 1) normal vector + one point on plane.

② parametrization 1) 2 vectors on plane. + one point on plane.

$\Uparrow$ not paralle

Q : Let n be a positive integer and $a_1, \dots a_n > 0$

Show that :

$$\underbrace{\frac{n}{\frac{1}{a_1} + \cdots + \frac{1}{a_n}}}_{\text{harmonic mean}} \leq \underbrace{\frac{a_1 + \cdots + a_n}{n}}_{\text{arithmetic mean}}$$

Sol by Cauchy - Swartz Inequality, take * $\vec{a} = (\sqrt{a_1}, \dots, \sqrt{a_n})$

$\vec{b} = \left(\frac{1}{\sqrt{a_i}}, \dots \frac{1}{\sqrt{a_n}}\right)$

$\Downarrow$

$|\vec{a} \cdot \vec{b}| \leq |\vec{a}||\vec{b}|$

$$\left| \sum_{i=n}^{n} \sqrt{a_i} \cdot \frac{1}{\sqrt{a_i}} \right| \leq \sqrt{\sum_{i=1}^{n} (\sqrt{a_i})^2} \cdot \sqrt{\sum_{i=1}^{n} \left(\frac{1}{\sqrt{a_i}}\right)^2}$$

squaring both sides.

$$n^2 \leq (a_1 + \cdots + a_n)\left(\frac{1}{a_1} + \cdots + \frac{1}{a_n}\right).$$

$$\therefore \frac{n}{\frac{1}{a_1} + \cdots + \frac{1}{a_n}} \leq \frac{(a_1 + \cdots + a_n)}{n} .$$

#.

Q : True or False.

Let $n > 0$ be an integer

1) A subset of $\mathbb{R}^n$ is either open or closed.

2) A subset of $\mathbb{R}^n$ cannot be both open and closed.

3) If A, B are bounded subsets of $\mathbb{R}^n$, then so is $A \cup B$

4) If A, B are connected subsets of $\mathbb{R}^n$, then so is $A \cup B$

5) There exists a subset A of $\mathbb{R}^n$ such that $\mathrm{Int}(A) = \phi$ and $\partial A \neq A$

Answer : 1) F

2) F

3) T

4) F

5) T

Q : Let $f(x,y,z) = \begin{cases} \dfrac{xyz}{x^3+y^4+z^4}, & \text{if } (x,y,z) \neq (0,0,0) \\ 0, & \text{if } (x,y,z) = (0,0,0) \end{cases}$

$(0,0,0)$

1) Show that f is discontinuous at $(0,0,0)$

2) Find $f_x(0,0,0)$ if it exists.

3) Is f_x continuous at $(0,0,0)$? Explain.

Sol : 1) $\lim\limits_{(x,y,z)\to(0,0,0)} f(x,y,z) = \lim\limits_{t\to 0} \dfrac{t^3}{t^3+t^4+t^4} = 1 \neq f(0,0,0) = 0$

$\therefore$ f is not continuous at $(0,0,0)$.

2) $f_x(0,0,0) = \lim\limits_{h\to 0} \dfrac{f(h,0,0) - f(0,0,0)}{h} = \lim\limits_{h\to 0} \dfrac{0-0}{h} = 0$

3). For $(x,y,z) \neq (0,0,0)$ $\quad f_x(x,y,z) = \overbrace{\dfrac{yz}{x^3+y^4+z^4} - \dfrac{3x^2yz}{(x^3+y^4+z^4)^2}}^{*}$ $\Rightarrow f_x = \begin{cases} *, & (x,y,z)\neq 0 \\ 0, & (x,y,z)=(0,0,0) \end{cases}$

$\lim_{\substack{(x,y,z)\to(0,0,0) \\ x=0, \quad y=z}} f_x(x,y,z) = \lim_{t\to 0} \frac{t^2}{t^4+t^4} = +\infty \neq f_x(0,0,0) \overset{\text{"}}{}\hspace{-1em}{}^{0}$

$\Rightarrow$ f_x is not continuous at $(0,0,0)$.